GEOMATICS ENGINEERING DEPARTMENT

SECOND YEAR GEOMATICS

GEODESY 2 (GED209)

LECTURE NO: 11

TWO AND THREE-DIMENSIONAL GEODETIC COMPUTATIONS

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Overview Of Previous Lecture

- Definition of Geodetic Datum
- What is meant by "Best Fitting"?
- Best Fitting Datum and how to achieve it in practice
- Notes on Establishment of Best Fitting Datum
- Significance of Accurate Geodetic Datum
- Summary

Overview Of Today's Lecture

Gravity Dependent Observations

Adjustment Computations In Geodesy

Two-Dimensional Geodetic Computations

Three-Dimensional Geodetic Computations

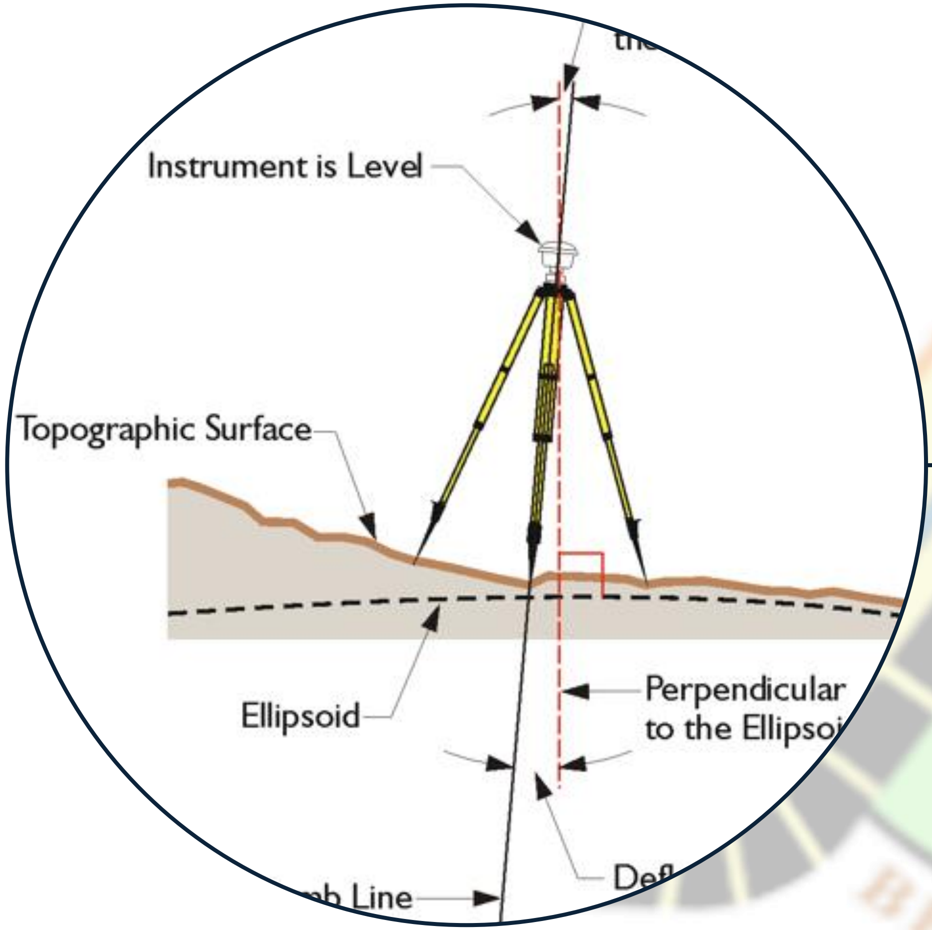
Summary

Expected Learning Outcomes

- Understand various gravity dependent observations.
- Acquire the ability to perform computations in two-dimensional/three-dimensional geodesy.
- Develop skills in performing adjustment computations, which involve the estimation of unknown quantities based on observations and statistical analysis.
- Learn about least squares adjustment methods and their applications in geodesy, including the adjustment of geodetic networks and the determination of accurate coordinates.

Geodetic Measurements

Gravity Dependent Observations



Astronomical observations

- Longitudes
- Latitudes
- Azimuths
- Horizontal Directions
- Zenith Distances
- Spirit Levelling

All the angles observed by a theodolite are measured when it is leveled in such a way that its vertical axis lies in the direction of the gravity vector

Gravity Dependent Observations

- Since the computations are carried out on the surface of a reference ellipsoid, then actual observations should be replaced by fictitious ones based on the direction of the normal to the ellipsoid.
- The replacement of the actual, the corresponding ellipsoidal one knows observations by the fictitious ones, ellipsoidal, is known as the reduction of the actual observations.
- Consequently, measured angles are reduced to angles between two planes containing the ellipsoidal normal and zenith distances are reduced to the equivalent ellipsoidal one.

Gravity Dependent Observations

- At stations where astronomical observations have been made to satisfy Laplace equation.
- It is important to notice that the reduction must be done through heights above ellipsoid.
- After applying these reductions to all observations then a least-squares adjustment, either by using the method of correlates or by the variation of coordinates method.

Summary

Astronomical observations

Reduction to ellipsoid

Geodetic computation

Adjustment

Adjustment Computations

- A least-squares adjustment, either by using the method of correlates or by the variation of coordinates method.

Two-dimensional	Three-dimensional

Two-Dimensional Geodetic Computations

(1) Two-Dimensional Geodetic Computations

- The network is adjusted in a two-dimensional frame consisting of latitudes and longitudes coordinate system defined on the surface of the reference ellipsoid.
- The observation point is assumed to lay on the normal to the ellipsoid through the adjusted geodetic position.
- The complete definition of this point with respect to the reference surface requires a third coordinate, which is the ellipsoidal height.
- This is obtained from a separate adjustment of the trigonometrical levelling data observed for the network, this adjustment usually ignores the distinction between the elevations above mean sea level determined by spirit levelling which are more nearly orthometric heights, heights above geoid, and ellipsoidal heights, heights above the reference ellipsoid.

(1) Two-Dimensional Geodetic Computations – Major Defects

1. Forcing the horizontal angles to fit the Laplace azimuths may alter the preliminary positions enough to require a reassessment of the astrogeodetic deflections. The consequences of this may include changes in the Laplace azimuths, making necessary reiteration of the adjustment.

2. Distances reduced to the geoid may differ from the corresponding ellipsoidal distances, an error of 6 meters in the geoid separation results in an error of *1 p.p.m.* in the base line. In a small country the difference may be small, but in a continental area it may be large especially if an old and ill-fitting ellipsoid is in use.

(1) Two-Dimensional Geodetic Computations – Major Defects

1. The corrected Laplace azimuths are burdened with the observational errors in astronomical latitudes, longitudes and azimuths, and accumulated errors in the geodetic survey between the origin and the observation point. Accordingly fixing these values during the adjustment procedure will affect the final adjusted values.
2. The observed astronomical latitudes and longitudes are not permitted to influence the final adjusted positions of the network stations.
3. The method cannot be adopted to permit the treatment of observed vertical angles and other levelling data simultaneously with the horizontal measurements.

Three-Dimensional Geodetic Computations

(2) Three-Dimensional Geodetic Computations

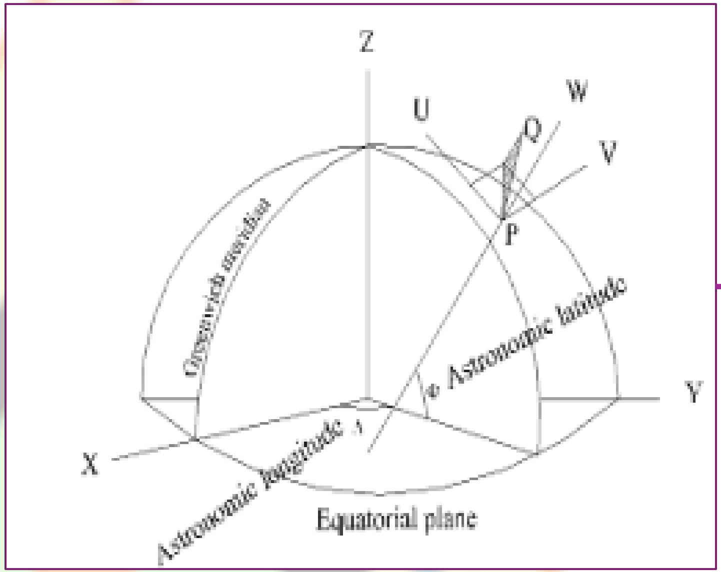
- All the observed quantities horizontal angles, distances, vertical angles, spirit levelling, astronomical latitudes, longitudes, and azimuths are combined in a single adjustment process.
- In other words, combining the horizontal and vertical adjustment of the network in one adjustment process.
- Accordingly, we can no longer deal with the two-dimensional coordinate system as it will not provide us with a quite adequate reference frame for the adjustment and consequently, we need a three-dimensional Cartesian system of coordinates upon which all the computations and adjustments are related.

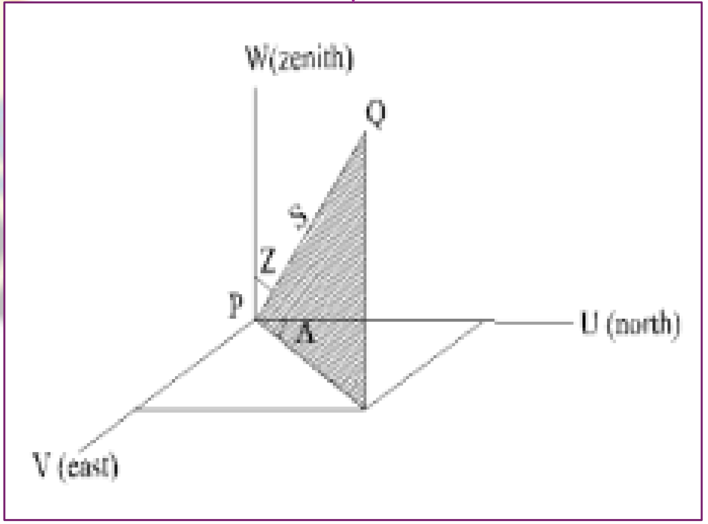
(2) Three-Dimensional Geodetic Computations – Formulation

- Bruns 1878 established the basic and fundamental idea of the rigorous computations. The main observational data used were, horizontal angles, zenith distances, spatial distances, astronomical observations for latitudes, longitudes, and azimuths.
- Determining the position of any point P requires five parameters, three of which are X,Y,Z coordinate of the point based on a rectangular system, and the other parameters are the astronomical longitude and latitude of the point, which defines the direction of the plumb line.

(2) Three-Dimensional Geodetic Computations – Formulation

- These observations are modeled in a local coordinate system U,V,W.
- The origin is at the observation station P, the W axis coincides with the plumb line, while the U and V axes are pointing northward and eastward respectively.





(2) Three-Dimensional Geodetic Computations – Formulation

- The azimuth A, and the zenith distances Z, to a neighbor station Q, distances S, from the observation station P. is given by: -

$$\tan A = \frac{v}{u} \qquad (1)$$

$$\cos Z = \frac{w}{s} \qquad (2)$$

$$S = \sqrt{u^2 + v^2 + w^2} \qquad (3)$$

Taking ΔX as the vector leading from P to Q in a three-dimensional coordinate system: -

$$\Delta X = \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} = \begin{vmatrix} X_Q & X_P \\ Y_Q & Y_P \\ Z_Q & Z_P \end{vmatrix} \qquad (4)$$

Then,

$$u = (\Delta X)^T e'$$
$$v = (\Delta X)^T e'' \qquad (5)$$
$$w = (\Delta X)^T n$$

Where e', e'', and n are the unit coordinate vectors in the UVW-system.

(2) Three-Dimensional Geodetic Computations – Formulation

- Inserting the values of u, v, w from Equation 5 into Equations 1, 2, and 3: -

$$\tan A = \frac{-\Delta X \sin \Lambda + \Delta Y \cos \Lambda}{-\Delta X \sin \Phi \cos \Lambda - \Delta Y \sin \Phi \sin \Lambda + \Delta Z \cos \Phi}$$

$$\cos Z = \frac{\Delta X \cos \Phi \cos \Lambda + \Delta Y \cos \Phi \sin \Lambda + \Delta Z \sin \Lambda}{\sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}} \qquad (6)$$

$$S = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

To employ least squares adjustment on Equation 6, approximate values of observations are preferred. Therefore, only small corrections are computed as follows: -

$$\delta A = a_1\, \delta X_P + a_2\, \delta Y_P + a_3\, \delta Z_P + a_4\, \delta X_Q + a_5\, \delta Y_Q + a_6\, \delta Z_Q + a_7\, \delta \Phi_P + a_8\, \delta \Lambda_P$$

$$\delta Z = b_1\, \delta X_P + b_2\, \delta Y_P + b_3\, \delta Z_P + b_4\, \delta X_Q + b_5\, \delta Y_Q + b_6\, \delta Z_Q + b_7\, \delta \Phi_P + b_8\, \delta \Lambda_P \qquad (7)$$

$$\delta S = c_1\, \delta X_P + c_2\, \delta Y_P + c_3\, \delta Z_P + c_4\, \delta X_Q + c_5\, \delta Y_Q + c_6\, \delta Z_Q$$

(2) Three-Dimensional Geodetic Computations – Formulation

- Equation 7 contains the main principles of three-dimensional geodesy. The correction can be done either in X, Y, Z system.
- The computations are done by taking for example , a preliminary values for φ, λ, h to be the same as Φ, Λ, and H for the corresponding points.
- These preliminary coordinates are converted to rectangular coordinates X, Y, and Z.
- A sufficient number of Equation 7 can then be solved by a least-squares adjustment procedure, for the unknowns $\delta\varphi$, $\delta\lambda$, δh, $\delta\Phi$, and $\delta\Lambda$.
- The parameters $\delta\Phi$, and $\delta\Lambda$ obtained by this process are really estimates of ξ and $\eta \sec\varphi$, but are burdened by errors in the preliminary values of Φ, and Λ which are usually the observed values.
- This method is presented by Wolf 1963a.

NOTES ON THREE-DIMENSIONAL GEODETIC COMPUTATIONS

Notes on Three-Dimensional Geodetic Computations

- Hotine 1969, suggests that if astronomical observations of Φ and Λ have not been made at some station in the network, approximate values obtained by interpolating ξ and η from other astrogeodetic stations in the area should be submitted.
- Stolz 1972 used gravimetrically determined deflection of the vertical only at points where astronomical observations of latitudes and longitudes had not been observed.
- Fubara 1972 adopted a similar model for the study of the requirements for successful three-dimensional adjustment, using first a simulated net of 15 stations and then a real 6 stations network.

NOTES ON THREE-DIMENSIONAL GEODETIC COMPUTATIONS

- **Fubara 1972 concluded that: -**

1. Precise astronomical latitudes should be observed at each station on the net.
2. No significant differences between the adjustment in curvilinear and Cartesian coordinates.
3. Because of the low precision on the vertical angles used, the geodetic heights were the most weakly determined. The remedial efficiency of including spirit leveled heights, and gravity observations were recognized and are being investigated.

End Of Presentation

Thank You For Attention!